Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Further Mathematics

Advanced
Further Mathematics Option 1
Paper 3: Further Mechanics 1
Further Mathematics Option 2
Paper 4: Further Mechanics 1

Sample Assessment Material for first teaching September 2017

Time: 1 hour 30 minutes

Paper Reference
9FM0/3C
9FM0/4C

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

S54442A

1/1/1/1/1/
S 5 4 4 4 2 A 0 1 2 5

Pearson

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A particle P of mass 0.5 kg is moving with velocity $(4\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $(2\mathbf{i} - \mathbf{j})\,\text{N s}$.

Show that the kinetic energy gained by P as a result of the impulse is 12 J.

(6)

$I = m(v-u)$

$2i - j = 0.5(v - 4i - j)$

$4i - 2j = v - 4i - j$

$v = ai + bj$

$4i = ai - 4i$ $\quad\quad$ $-2j = bj - j$

$a = 8$ $\quad\quad$ $b = -1$

$\therefore\ v = 8i - j$

$|v| = \sqrt{8^2 + (-1)^2}$

$= \sqrt{65}\ \text{ms}^{-1}$

$|u| = \sqrt{4^2 + 1^2}$

$= \sqrt{17}\ \text{ms}^{-1}$

$\Delta KE = \frac{1}{2} m (v^2 - u^2)$

$= \frac{1}{2}(0.5)(65 - 17)$

$= \boxed{12\text{J}}$

Question 1 continued

(Total for Question 1 is 6 marks)

2. A parcel of mass 5 kg is projected with speed $8\,\text{m s}^{-1}$ up a line of greatest slope of a fixed rough inclined ramp.

The ramp is inclined at angle α to the horizontal, where $\sin\alpha = \frac{1}{7}$

The parcel is projected from the point A on the ramp and comes to instantaneous rest at the point B on the ramp, where $AB = 14\,\text{m}$.

The coefficient of friction between the parcel and the ramp is μ.

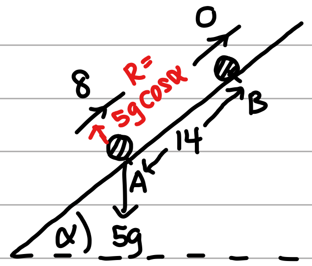
In a model of the parcel's motion, the parcel is treated as a particle.

(a) Use the work-energy principle to find the value of μ.

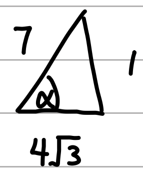
(5)

(b) Suggest one way in which the model could be refined to make it more realistic.

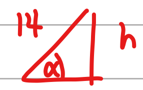
(1)



$\sin\alpha = \frac{1}{7}$



$\therefore \cos\alpha = \frac{4\sqrt{3}}{7}$



at A : KE $= \frac{1}{2}(5)(8^2)$
$= 160$
GPE $= 0$

at B : KE $= \frac{1}{2}(5)(0)$
$= 0$
GPE $= 5g\left(\frac{1}{7} \times 14\right)$

$\sin\alpha = \frac{h}{14}$
$h = \sin\alpha \times 14$

wd by friction : $\mu R \times d = \mu(5g\cos\alpha)(14)$
$= 70g\left(\frac{4\sqrt{3}}{7}\right)\mu$
$= (40g\sqrt{3})\mu$

$160 = 10g + (40g\sqrt{3})\mu$
$\therefore \mu = \frac{160 - 10g}{40g\sqrt{3}}$
$= 0.091315\ldots$
$\boxed{\mu \simeq 0.091}$

b) air resistance could be modelled as a function of velocity

Question 2 continued

(Total for Question 2 is 6 marks)

3. A particle of mass m kg lies on a smooth horizontal surface.

Initially the particle is at rest at a point O between two fixed parallel vertical walls.

The point O is equidistant from the two walls and the walls are 4 m apart.

At time $t = 0$ the particle is projected from O with speed u m s^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{3}{4}$

The magnitude of the impulse on the particle due to the first impact with a wall is λmu N s.

(a) Find the value of λ.

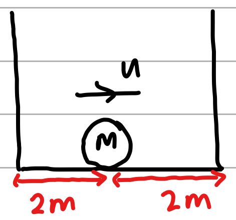
(3)

The particle returns to O, having bounced off each wall once, at time $t = 7$ seconds.

(b) Find the value of u.

(5)

a)



$e = \frac{3}{4}$

$I = m(v - u) \qquad v = ue = \frac{3}{4}u$

$\lambda mu = m\left(\frac{3u}{4} - -u\right)$

$\lambda u = \frac{7u}{4} \qquad \boxed{\therefore \lambda = \frac{7}{4}}$

b) until first collision : $s = ut$

$2 = ut$

$t_1 = \frac{2}{u}$

until second collision : $s = ut_2$

$4 = \frac{3}{4}ut_2$

$t_2 = \frac{16}{3u}$

until it returns to O : $s = ut_3$

$2 = \frac{9}{16}ut_3$

$t_3 = \frac{32}{9u}$

Question 3 continued

$$t_1 + t_2 + t_3 = \frac{2}{u} + \frac{16}{3u} + \frac{32}{9u}$$

$$= \frac{1}{u}\left(2 + \frac{16}{3} + \frac{32}{9}\right)$$

$$= \frac{98}{9}\left(\frac{1}{u}\right)$$

$$\frac{98}{9}\left(\frac{1}{u}\right) = 7$$

$$7u = \frac{98}{9}$$

$$\boxed{u = \frac{14}{9}}$$

(Total for Question 3 is 8 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

4.

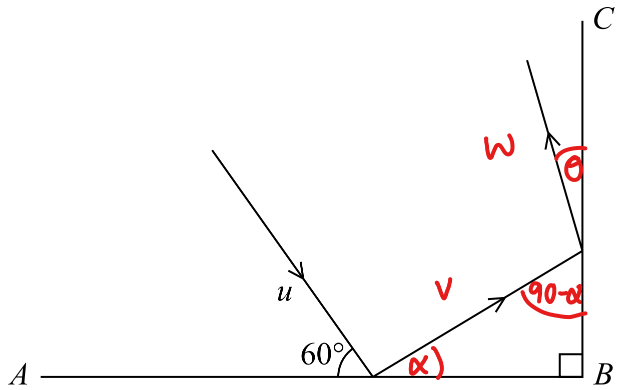


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC are perpendicular vertical walls.

The floor and the walls are modelled as smooth.

A ball is projected along the floor towards AB with speed $u\,\mathrm{m\,s^{-1}}$ on a path at an angle of 60° to AB. The ball hits AB and then hits BC.

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall AB is $\frac{1}{\sqrt{3}}$

The coefficient of restitution between the ball and wall BC is $\sqrt{\frac{2}{5}}$

(a) Show that, using this model, the final kinetic energy of the ball is 35% of the initial kinetic energy of the ball.

(8)

(b) In reality the floor and the walls may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

(1)

a) $\overrightarrow{\text{C.L.M.}}$ $u\cos 60 = v\cos\alpha$ — ①

N.I.L : $ue\sin 60 = v\sin\alpha$

$\frac{u\sin 60}{\sqrt{3}} = v\sin\alpha$ — ②

①^2 + ②^2 : $v^2(\sin^2\alpha + \cos^2\alpha) = u^2\left(\frac{\sin^2 60}{3} + \cos^2 60\right)$

$v^2 = u^2\left(\frac{1}{4} + \frac{1}{4}\right)$

$= \frac{u^2}{2}$

$v = \frac{u}{\sqrt{2}}$

Question 4 continued

substitute $v = \frac{u}{\sqrt{2}}$ into ①

$$\Rightarrow \frac{\cancel{u}}{\sqrt{2}} \cos\alpha = \cancel{u} \cos 60$$

$$\cos\alpha = \sqrt{2} \cos 60$$
$$= \frac{\sqrt{2}}{2}$$

$$\alpha = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$
$$= 45°$$

for second wall...

$\uparrow$ C.L.M : $v\cos 45 = w\cos\theta$ — ①

N.L.L : $ve\cos\alpha = w\sin\theta$

$$v\left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{\frac{2}{5}}\right) = w\sin\theta \quad — ②$$

①^2 + ②^2 : $w^2(\sin^2\theta + \cos^2\theta) = v^2(\cos^2 45 + \frac{2}{5} \times \frac{2}{4})$

$$w^2 = \frac{7v^2}{10} = \frac{7u^2}{20}$$

initial KE $= \frac{1}{2}mu^2$

final KE $= \frac{1}{2}m\left(\frac{7u^2}{20}\right)$

final KE $= \frac{7}{20}$ of initial KE

hence final KE is 35% of initial KE

b) final speed would actually be lower as there is resistance, so our answer is too large.

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. A car of mass 600 kg is moving along a straight horizontal road.

At the instant when the speed of the car is $v\,\mathrm{m\,s^{-1}}$, the resistance to the motion of the car is modelled as a force of magnitude $(200 + 2v)$ N.

The engine of the car is working at a constant rate of 12 kW.

(a) Find the acceleration of the car at the instant when $v = 20$

(4)

Later on the car is moving up a straight road inclined at an angle θ to the horizontal,

where $\sin\theta = \dfrac{1}{14}$

At the instant when the speed of the car is $v\,\mathrm{m\,s^{-1}}$, the resistance to the motion of the car from non-gravitational forces is modelled as a force of magnitude $(200 + 2v)$ N.

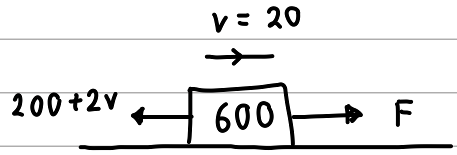
The engine is again working at a constant rate of 12 kW.

At the instant when the car has speed $w\,\mathrm{m\,s^{-1}}$, the car is decelerating at $0.05\,\mathrm{m\,s^{-2}}$.

(b) Find the value of w.

(5)

a)



$P = Fv$

$12000 = Fv$

$\dfrac{12000}{20} = F$

$F = 600\ \mathrm{N}$

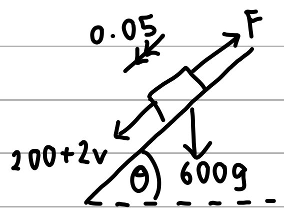
$\xrightarrow{+}$ N2L (car) : $F - 200 - 2v = 600a$

$600 - 200 - 2(20) = 600a$

$a = \dfrac{360}{600}$

$\boxed{a = 0.6\,\mathrm{ms^{-2}}}$

b)



$\sin\theta = \frac{1}{14}$

+ N2L (car) $\quad 600g\sin\theta + 200 + 2w - F = 600(0.05)$

$F = \dfrac{600g}{14} + 200 + 2w - 30$

$P = Fv$

$12000 = Fw \qquad \therefore\ F = \dfrac{12000}{w}$

Question 5 continued

$\times w$ $\quad \dfrac{12000}{w} - 590 = 2w$

$12000 - 590w - 2w^2 = 0$

$2w^2 + 590w - 12000 = 0$

by quadratic formula : $\dfrac{-590 \pm \sqrt{590^2 - 4(2)(-12000)}}{2(2)}$

$w = 19.1\ldots$ or $w = -314\ldots$

$w > 0$ so $\boxed{w = 19.1\ \mathrm{ms^{-1}}}$

(Total for Question 5 is 9 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

6. [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass $3m$ kg.

The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately before the collision the velocity of A is $(3\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-5\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$.

At the instant of collision, the line joining the centres of the spheres is parallel to **i**.

The coefficient of restitution between the spheres is $\frac{1}{4}$

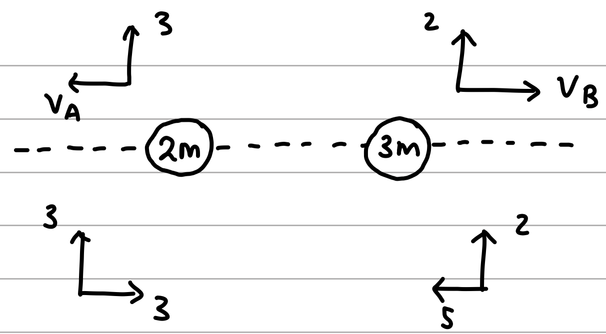
(a) Find the velocity of B immediately after the collision.

(7)

(b) Find, to the nearest degree, the size of the angle through which the direction of motion of B is deflected as a result of the collision.

(2)

a)



$\overrightarrow{\text{CLM}}$ $\quad 2m(3) - 3m(5) = 3m(V_B) - 2m(V_A)$

$-9 = 3V_B - 2V_A \quad$ — ①

$\overrightarrow{\text{NIL}}$: $\frac{1}{4} = \frac{V_A + V_B}{(5+3)}$

$2 = V_A + V_B$

$V_A = 2 - V_B$

① $\quad -9 = 3V_B - 2(2 - V_B)$

$= 5V_B - 4$

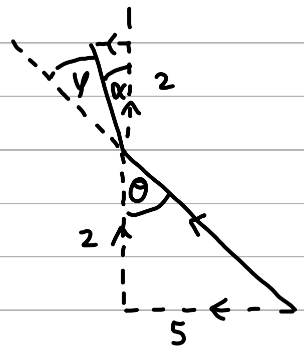
$5V_B = -5$

$V_B = -1$

hence velocity of B = $\boxed{-\mathbf{i} + 2\mathbf{j}}$

Question 6 continued

b)



$\tan\alpha = \frac{1}{2}$

$\alpha = \arctan\left(\frac{1}{2}\right)$

$\tan\theta = \frac{5}{2}$

$\theta = \arctan\left(\frac{5}{2}\right)$

γ = required angle $= \theta - \alpha$

$= \tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)$

$= 41.6°$

$\boxed{\doteq 42°}$

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 9 marks)

7. A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $3mg$.

The other end of the string is attached to a fixed point O on a ceiling.

The particle hangs freely in equilibrium at a distance d vertically below O.

(a) Show that $d = \frac{4}{3}a$.
(3)

The point A is vertically below O such that $OA = 2a$.

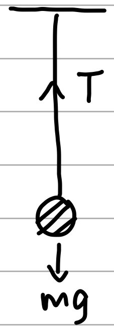
The particle is held at rest at A, then released and first comes to instantaneous rest at the point B.

(b) Find, in terms of g, the acceleration of P immediately after it is released from rest.
(3)

(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B.
(5)

(d) Find, in terms of a, the distance OB.
(3)

a)



$T = mg$

$\frac{\lambda x}{l} = mg$

$\frac{3\cancel{mg}(x)}{a} = \cancel{mg}$

$\frac{3x}{a} = 1$

$x = \frac{a}{3}$ $\therefore d = a + \frac{a}{3}$

$= \boxed{\frac{4a}{3}}$

b) $\frac{3\cancel{m}g(\cancel{a})}{\cancel{a}} - \cancel{m}g = \cancel{m}\ddot{x}$

$\ddot{x} = 3g - g$

$\boxed{\ddot{x} = 2g}$

c) Final EPE $= \frac{\lambda x^2}{2l}$

$= \frac{3mg(a^{\cancel{2}})}{2\cancel{a}}$

$= \frac{3}{2}mga$

Initial EPE $= \frac{\cancel{3}mg\left(\frac{a}{3}\right)^{\cancel{2}}}{2\cancel{a}}$

$= \frac{mga}{6}$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 7 continued

$\frac{3}{2}mga = \frac{1}{6}mga + \frac{1}{2}mv^2 + mg\left(\frac{2a}{3}\right)$

$\frac{1}{2}mv^2 = \frac{2}{3}mga$

$\frac{1}{2}v^2 = \frac{2}{3}ga$

$v^2 = \frac{4}{3}ga$

$\boxed{v = \sqrt{\frac{4ga}{3}}}$

d) at B, $v=0$ so $KE = 0$

EPE lost = GPE gained

$\frac{3mga^2}{2a} = mgh$

$h = \frac{3}{2}a$

$OB = 2a - \frac{3}{2}a$

$\boxed{OB = \frac{a}{2}}$

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 14 marks)

8. A particle P of mass $2m$ and a particle Q of mass $5m$ are moving along the same straight line on a smooth horizontal plane.

They are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is $2u$ and the speed of Q is u.

The direction of motion of Q is reversed by the collision.

The coefficient of restitution between P and Q is e.

(a) Find the range of possible values of e.

(8)

Given that $e = \frac{1}{3}$

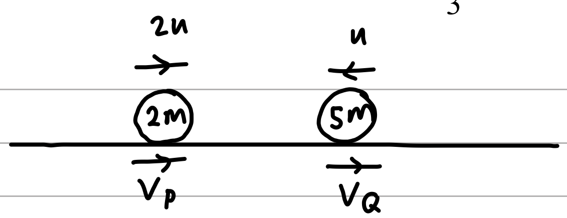
(b) show that the kinetic energy lost in the collision is $\frac{40mu^2}{7}$.

(5)

(c) Without doing any further calculation, state how the amount of kinetic energy lost in the collision would change if $e > \frac{1}{3}$

(1)

a)



$\overrightarrow{\text{CLM}}$ $\quad 2m(2u) + 5m(-u) = 2m(V_P) + 5m(V_Q)$

$-u = 2V_P + 5V_Q \quad — ①$

$\overrightarrow{\text{NIL}}$: $e = \frac{V_Q - V_P}{3u}$

$\therefore\ 3ue = V_Q - V_P$

$V_P = V_Q - 3ue$

$\downarrow$

① : $-u = 2V_Q - 6ue + 5V_Q$

$7V_Q = 6ue - u$

$V_Q = \frac{u}{7}(6e - 1)$

but Q changes direction so $V_Q > 0$

$\Rightarrow \frac{u}{7}(6e-1) > 0$

$6e - 1 > 0$

$e > \frac{1}{6}$ $\qquad$ so... $\boxed{\frac{1}{6} < e \leq 1}$

Question 8 continued

b) $V_Q = \frac{u}{7}(1)$

$V_P = \frac{u}{7} - 3u\left(\frac{1}{3}\right)$

$= \frac{u}{7} - u$

$= \frac{-6u}{7}$

KE after: $Q: \frac{1}{2}(5m)\left(\frac{u}{7}\right)^2 = \frac{5mu^2}{98}$

$P = \frac{1}{2}(2m)\left(\frac{6u}{7}\right)^2 = \frac{36mu^2}{49}$

$\boxed{\frac{11mu^2}{14}}$

KE before: $Q: \frac{1}{2}(5m)u^2 = \frac{5mu^2}{2}$

$P: \frac{1}{2}(2m)(2u)^2 = 4mu^2$

$\boxed{\frac{13mu^2}{2}}$

$\Delta KE = \frac{11mu^2}{14} - \frac{13mu^2}{2}$

$= \frac{-40}{7}mu^2$ $\therefore$ lost in KE $= \frac{40}{7}mu^2$

c) $e > \frac{1}{3}$ $\rightarrow$ less energy lost in the collision as it is more elastic so there would be less KE lost

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS